Calibration of the Short period seismometer using close by Broad Band seismometer installation and seismic noise recordings

V. Pinsky, T. Meirova, A Hofstetter
Seismological division of the Geophysical Institute of Israel

Abstract

We address short period (SP) seismometer calibration problem using reference broad-band (BB) seismometer standing close to it. By instrument correction of the BB seismometer record we restituted the ground motion and used it as input to the SP seismometer. Then using the “input” and the “output” recordings of the SP seismometer we estimated it's frequency response using ARMA parametric representation, the grid-search method for direct search of amplification, damping and eigen frequency; and non-parametric spectral ratio method. The ground motion of the SP seismometer was then estimated using the inverse transfer function. The method was successfully tested using experimental installation of close by Trillium 40 seismometer and the SP seismometers S13 and L4C using simultaneous noise recordings and using noise and signal recordings of the STS-2 and the S13 seismometer at the MMLI station site of the Israel Seismic Network. While using ARMA parameters estimated using 1 minute noise recordings the ground motion residual of instrument correction appeared to be less than 10%.

Introduction

The basic instrument of seismology is the seismometer. Any quantitative analysis of seismic waveform demands that the frequency response of the seismic instrument that recorded the data of interest be known to some degree of precision. The following are the two major, well-established techniques for establishing absolute calibration of a seismometer: (1) shake tables and (2) electromagnetic calibrator method for determining seismometer response. Shake tables provide, in principle, the most direct calibration measurement currently available, since ground motion is directly measured for comparison with the voltage output produced by the sensor. However, they are massive, expensive devices that are difficult to construct and maintain. Consequently, electromagnetic calibrator method is the most common, alternative way of determining the response of a seismometer. Previously it was fulfilled using calibration coils. Now signal coils are mostly used (Rodgers et al, 1995) providing almost the same effect. However, the motor constant of the electromagnetic calibrator must be known and provisions must be made for carefully measuring the peak-to-peak current through the coil (PORTABLE SHORT-PERIOD SEISMOMETER MODEL S-13J VERTICAL AND HORIZONTAL OPERATION AND MAINTENANCE MANUAL, 2000-2002). Besides, it is not based on ground motion but rather an artificial perturbation of the internal mechanism of the seismometer. The technique presented here makes it possible to calibrate seismometers using noise and
signal recordings of the calibrated reference seismometer (BB or calibrated SP seismometer) standing nearby without any additional knowledge or information. There were few publications devoted to this problem: (Pavli's & Frank, 1994), developed robust estimation procedure based on so-called M-estimates and reported a successful calibration of different types of seismometers in a broad working frequency range. The advantage of the method over the electric calibration that it's result can be immediately viewed in terms of estimated ground motion compared to the ground motion of the reference seismometer.

**Problem description**

Using BB recordings of noise and signal obtain Frequency response of the SP, standing close by.

The scheme of the deployment pictures in Figure 1.

![Figure 1. Scheme of the deployment include BB (Trillium 40) seismometer, SP - S13 or/and L4C seismometer. Trident includes A/D and frequency independent amplification.](image)

**PLAN.**

1. Using BB recordings "A" make instrument correction to estimate ground motion X.

2. Using X and the SP output Y estimate parametric ARMA model of the SP seismometer and draw it's estimated frequency response "EH" (Amplitude and Phase)

3. Compare EH with standard SH of the seismometer.

4. Compare input X with estimated input EX= Y*EH⁻¹
5. Find $Z$ (discrete poles) of the system

6. Find $S$ (continuous) poles and zeros of the system (Tustin's method)

7. Use simultaneous recordings of SP and BB at the seismometer site.

I. ESTIMATION OF ARMA MODEL FOR THE SP SEISMOMETER

II.1. Using BB recordings make instrument correction to obtain ground motion $"X"$

Table 1. Parameters of the Trillium 40 seismometer transfer function

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Nominal Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_m$</td>
<td>Zeros</td>
<td>0</td>
<td>rad/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>−68.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>−323</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>−2530</td>
<td></td>
</tr>
<tr>
<td>$p_m$</td>
<td>Poles</td>
<td>−0.1103 ±0.1110i</td>
<td>rad/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−86.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>−241 ±178i</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>−535 ±719i</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Normalization factor</td>
<td>$1.104 \times 10^5$</td>
<td>(rad/s)$^2$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Normalization frequency</td>
<td>1</td>
<td>Hz</td>
</tr>
<tr>
<td>$S$</td>
<td>Ground motion sensitivity at $f_0$</td>
<td>1553</td>
<td>V s/m</td>
</tr>
</tbody>
</table>
Figure 2. Bode plot for Trillium 40 seismometers. The seismometer transfer function to ground motion (red).

We were using recordings of the two types: 1) the 24 hours long continuous recordings from the experimental installation of a new Trillium 40 seismometer, and the L4C and the new S13 seismometer; and 2) the trigger data from the MMLI station where in 2007-2008 there was a parallel installation of the BB STS-2 seismometer and the S13 seismometer. First we shall consider the experimental data.

We are using coefficients $k$, $S$ and poles and zeros specification for the Trillium 40 BB seismometer (Table 1):

The seismometer sensitivity, poles, and zeros define the transfer function

According to this equation:

$$ F(S) = C \frac{(S-Z_n)}{(S-P_n)} $$

(1)

The transformation coefficient for the seismometer $C$ (from counts to velocity (m/s)) $\text{Vel} = C \ast \text{counts}$ is calculated according formula: $C = \frac{\text{V}}{(R \ast S \ast k)}$
where A/D resolution $R=2^{24}=1677216$ counts, sensitivity $S$ for the Trillium 40 seismometer equal to 1553 V/(m/s) and for the S13 seismometer $S=629$ V/(m/s). For the L4C seismometer it depends on the choice of the resistor in the outer circuit. The voltage dynamic range $V$ for the Trillium was chosen $U=40$ V and for the S13 and for L4C $U=8$ V. Consequently for Trillium 40 the constant $C_T=1.34\times10^{-13}$, and for the S13 (L4C) $C_S=7.578\times10^{-9}$.

Figure 3. Four seconds fragment of ground motion $X$-(green) restituted using Trillium 40 instrument correction, output of the S13 seismometer $Y$- (red) signals recorded on October 20 06:55:40 (Start=25000 s, End=25004 s) as function of velocity (nm/s) vs. sample number $= h^*(T\text{-Start})$, $h=40$ sample rate, deviation $X-Y$ is shown by blue.

The Trillium 40 signal after instrument correction (and normalization using constant $C_T$) and High-path 2-pole (HP) filtering from 0.1 Hz was taken as input signal $X$ of the Linear Dynamic System (SP seismometer). The output $Y$ is the HP filtered observed output signal of the S13 seismometer normalized using constant $C=7.578\times10^{-9}$.

II.2 Using $X$ and the SP output $Y$ estimate ARMA model of SP and draw it's estimated frequency response "EH"

According to differential equation of a suspended pendulum damped movement continuous time frequency transfer function of a seismometer is determined by:
\[ T_c(f) = \frac{S^2}{(S-P_1)(S-P_2)}. \]  

where \( S = i2\pi f, \) \( P_1 \) and \( P_2 \) are the poles: \( P_1 = P_2 = (D \pm i\sqrt{(1-D^2)})2\pi f_0, \) \( D = 0.707 \) critical damping: \( \sqrt{(1-D^2)} = \pm D, \) \( f \) - frequency, \( f_0 \) - eigen frequency of seismometer. For example, \( f_0 = 1 \) Hz for the S13 and L4C seismometer and \( f_0 = 0.0083 \) Hz for the STS-2. The poles for the S13 and the L4C seismometers are \( P_{1,2} = -4.44 \pm i4.44. \)

Because our observations are digitized seismograms, for estimation of the SP seismometer transfer function we are using discrete time representation of Linear Dynamic System in a form or the ARMA model:

\[ Y_t = \sum a_k Y_{t-k} + \sum b_m X_{t-m+1}, \]

where \( a_k, b_m \) autoregression and moving average coefficients respectively (ARMA).

The corresponding frequency transfer function of the system (1) is equal polynomial ratio:

\[ T_d(Z) = \sum b_m Z^{-m+1}/[1 - \sum a_k Z^{-k}] \]

where \( Z = \exp(i2\pi f). \)

The ARMA coefficients are computed from continuous transfer function (1) using Tustin’s substitution method: (ex. Scherbaum, 2002)

\[ S = (1-Z^{-1})(1+Z^{-1})/2/h, \]

where \( h \) - sampling time interval, \( h = 40 \) smpl/s. As the result for the SP seismometer (S13 or L4C (shunt resistance \( R = 8904 \) ) with \( f_0 = 1 \) \( D = 0.707 \) we get

\[ a1 = 1.7791092, \ a2 = -0.80119419 \]
\[ b1 = 0.89507586, \ b2 = -1.7901517, \ b3 = 0.89507586 \]
These coefficients are used as first approximation for the ARMA model estimated by maximum likelihood algorithm of Pinsky, (1986) $a_k$, $b_m$, providing optimal ARMA estimators. For the given X 10000 samples (250 sec) and the corresponding Y (Figure 3) the estimated ARMA coefficients for the S13 seismometer are:

\[a_1=1.7779169 \quad a_2=-0.79432166 \]
\[b_1=0.896047 \quad b_2=-1.8000894 \quad b_3=0.904727\]

And for the L4C seismometer ARMA coefficients are almost the same:

\[a_1 = 1.7565125 \quad a_2 = -0.77628589 \]
\[b_1 = 0.89145815 \quad b_2 = -1.7903472 \quad b_3 = 0.89919144\]

II.3. Compare EH with standard SH of the seismometer

Figure 4 depicts theoretical and estimated frequency amplitude and phase transfer functions of the S13 seismometer using X input and Y output as shown in Figure 3.
II.4. Compare input X with estimated input \( EX = Y^*E_1 \)

From (2) we get

\[
X_t = Y_t/b_0 - \sum a_k / b_0 \; Y_{t-k} + \sum b_m / b_0 \; X_{t-m+1} = \sum A_m \; X_{t-m} + \sum B_k \; Y_{t-k+1}
\] (6)

where

\( A_m = b_{m+1}/b_1, \; m=1,M, \; B_1 = 1/b_1, \; B_k = -a_{k-1}/b_1, \; k=2,K \)
Figure 5. (a) 24 hours X (Green) estimated input for the S13 and the L4C seismometers (Trillium 40 remove instrument ground motion), EX (red) estimated input using equation (2), residual X-EX (blue). Normalized RMS=0.09, (b) 4 seconds fragment starting at second 25000.

The result of filtration of the 24 hours record of the S13 seismometer made on 20.10.2011 is shown in figure 5 together with remove instrument Trillium 40 recording and the difference between the traces, which has 9% relative RMS. For the L4C the result is even better with 7% RMS.

II.5. Find Z (discrete) poles and zeros of the system

The poles of the discrete system (2) are found as roots of the AR polynomial with estimated AR coefficients:
\[ P_a(Z) = 1 - a_1 Z - a_2 Z^2 = 0, \]

(7)

\[ Z_{12} = (-a_1 \pm \sqrt{a_1^2 + 4a_2})/(2a_2) \]

For the S13 seismometer we get: \( P_{12} = 1.119141646 - 0.08036049i \),
For the L4C seismometer: \( P_{12} = 1.131356709 - 0.090648792i \)

Figure 6 shows positions of the poles relative R=1 circle in the complex numbers plane.
Nulls are determined using estimated MA coefficients from

\[ P_b(Z) = b_1 + b_2 Z + b_3 Z^2 = 0 \]

For S13: \( Z_{12} = 0.995 \pm 0.027 i \)
For the L4C: \( Z_{12} = 0.976 \pm 0.01778i \)

**Figure 6.** Poles of the estimated ARMA model.

### II.6. Find S (continuous) poles and zeros of the system (Tustin's method)

The continuous system poles and zeroes are obtained using Tustin's method (5). Thus, for S13 continuous poles are estimated as \( P_{12} = 4.606 \pm 2.859 i \) and zeros as \( Z_{12} = -0.186 \pm 1.085 i \)
and for the L4C as \( P_{12} = 5.066 \pm 3.187 i \) and zeros \( Z_{12} = -0.965 \pm 0.729 i \)

From the poles (eq. (2)) expression as functions of damping D and eigen frequency \( F_0 \) for the S13 seismometer we get \( D = 0.72 \) and \( F_0 = 1.015 \)
and for the L4C \( D = 0.768 \) and \( F_0 = 0.953 \). However, notice that zeros of the continuous system are not nulls (as in eq.(2)), thus D and \( F_0 \) computations might be wrong.
II. DETERMINATION of the SP SEISMOMETER TRANSFER FUNCTION USING OTHER SP SEISMOMETER AS A REFERENCE

From Figure 5 we can guess that ground motion obtained by remove instrument of the S13 (or other SP) seismometer can be used as input for estimation of transfer function of another SP seismometer. So we did with transfer function of the S13 seismometer obtained above and got the following ARMA estimates for the L4C seismometer with RMS = 0.095:

\[
a_1 = 1.7678236 \quad a_2 = -0.7822885 \\
b_1 = 0.89471465 \quad b_2 = -1.8137558 \quad b_3 = 0.91901922
\]

The corresponding amplitude and phase characteristics compared to theoretical are shown in Figure 7.

**Figure 7.** Estimates for amplitude and phase characteristics (red) of the L4C seismometer obtained using S13 as a reference station, compared to theoretical ones (green).
III. DETERMINATION of the SP SEISMOMETER TRANSFER FUNCTION AT THE SITE USING SIGNAL RECORDINGS.

We found out that within period 2007.01-2008.2 there we STS-2 BB seismometer together with the S13 seismometer at station MMLI and used the event trigger data for determining S13 transfer function using BB vertical channel data as input “X” and “S13” data as output “Y”. The results of the estimates are shown in the following figures. Figure 7 shows input X and output Y (event 200705030606 – magnitude $m_d=3.3$, Jordan-Valley).

The estimate of the transfer function using the whole recording (figure 8) is shown in Figure 9. ARMA coefficients are as follows:

AR-COEF: 1.7894694 -0.8124721

MA-COEF 0.89711887 -1.7903908 0.89282721

and slightly deviate from the theoretical ARMA coefficients for the S13 seismometer.
Figure 9. Amplitude (top) and phase (bottom) of the estimated (red) and theoretical (green) transfer functions, using effest algorithm.
Figure 10. Response $Z$ of the estimated linear system (green) to the impact $X$, observations $Y$ (red) and residual $Y-Z$ (blue) (top), signal fragment (middle) and noise fragment (bottom).
IV. ALTERNATIVE ESTIMATION METHODS

IV.1. Grid-search

We determine transfer function using formula (2) by a grid-search in the 3D space \((A,D,F0)\) where \(A\) - gain between \((0.2-1.5)\), \(D\) critical damping between \((0.3 - 0.95)\), and \(F0\) – eigen frequency \((0.5 – 2)\) splitting the intervals into 13, 20 and 100 parts respectively. The estimates, obtaines for the case above are:

\(A0=1.,\) \(D0=0.657,\) \(F0=0.995.\) Figure 11. shows the residual as function of eigen frequency when parameters \(A\) and \(D\) are fixed to optimal values

![Figure 11](image)

**Figure 11.** Residual \(S\) as function of eigen frequency for fixed \(D=0.657\) and \(A=1.\)

IV.2 Spectral ratio.

The amplitude transfer function is determines as ratio of smoothed amplitudes of the FFT transform of \(Y\) and \(X\):

\[ A(f) = \left| \frac{\text{FFT}Y}{\text{FFT}X} \right| \]

and phase transfer function as smoothed difference of phases of the FFTs:

\[ P(f) = \text{ARG}[(\text{FFT}Y) - \text{ARG}(\text{FFT}X)] \]

FFT is taken at consecutive time windows \((1024\) sample points\) along the trace
The results of the computations for the 16 consecutive time windows are shown in Figure 12. There is unfit between theoretical and estimated transfer function above 16 Hz, explained probably by the HP filtering of the data.

**Figure 12.** Smoothed (10 points) Amplitude Spectral ratio Y/X in 16 time windows and smoothed phase difference.
V. DETERMINATION of the SP SEISMOMETER TRANSFER FUNCTION AT THE SITE USING NOISE RECORDINGS.

We are taking noise part (first 97 seconds) of the recording above. By application of efest we are getting following ARMA coefficients, which are very close to those obtained above (and to the theoretical):

AR-COEFF. 1.7961994 -0.81795889
MA-COEFF. 0.89588112 -1.7905258 0.89444649

However, the RMS = 0.54, which is, probably attributed to electronic noise. The record and the computed response look as in Figure 13.

![Figure 13](image)

**Figure 13.** Response Z of the estimated linear system (green) to the impact X for the noise part of the record (the first 97 seconds), observations Y (red) and residual Y-Z (blue).

The estimates of the grid-search are D=0.657 and the F0=0.965.

The estimates of the spectral ratio for 7 consecutive windows of 512 points are shown in Figure 14.
Figure 14. Smoothed (10 points) Amplitude Spectral ratio Y/X and smoothed phase difference in 7 time windows (10 sec) of noise previous to the signal.
IV. RESTITUTION OF THE GROUND MOTION BY REMOVE INSTRUMENT.

For ground motion restitution we first provide BP (0.2-16 Hz) filtering of the BB and SP seismometers data X and Y respectively, make FFT of FX and FY, compute $W = FX \cdot T_d^{-1}$, where $T_d$ is computed using (4) with ARMA coefficients, determined in section III, and, finally make inverse FFT of $W$.

The result of comparison between the ground motion $X$ (BP filtered) and the estimate is shown in

![Comparison between ground motion and restituted signal](image)

**Figure 15.** Comparison between ground motion (red) and the restituted signal (green) for the 410 seconds time period (signal and noise) (b) for the zoom of the noise section and (c) for the zoom of the signal section. The residual is shown by blue line. Total RMS=0.076.

**CONCLUSIONS**

- Using simultaneous recordings of the Trillium 40 and SP seismometers S13 and L4C at a test site and the the STS-2 and S13 seismometers at MMLI station site we have shown that
calibration of the SP seismometer is possible by estimation of ARMA parameters of the Linear Dynamic Discrete System model of the SP seismometer.

- It is shown that for the SP seismometer calibration it is possible using a calibrated SP seismometer standing nearby as a reference. It is much more practical and cheaper than using a BB seismometer, requiring a long time for calming down and strict isolation from temperature and atmospheric pressure influence.

- The high precision ARMA estimates achieved are almost equivalent for both the seismic noise and the seismic signal, provided there is an adequate covering of the seismometer frequency band-path.

- Provided high quality of the seismometers installation it is possible to achieve accuracy of the ground motion restitution within 10% error of the residual RMS.

- Alternative methods presented of the grid-search for the gain, damping and eigen frequency and of spectral ratio in a set of time windows showed to be practical for the SP seismometer calibration.

- The method might be recommended in all cases when calibration of the seismometer is impossible or difficult by using calibration device.

REFERENCES


PORTABLE SHORT-PERIOD SEISMOMETER MODEL S-13J VERTICAL AND HORIZONTAL OPERATION AND MAINTENANCE MANUAL. GEOTECH INSTRUMENTS, LLC 2000-2002